Novel soft-photon analysis of ppγ below pion-production threshold

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(February 9, 2008)

Abstract

A novel soft-photon amplitude is proposed to replace the conventional Low soft-photon amplitude for nucleon-nucleon bremsstrahlung. Its derivation is guided by the standard meson-exchange model of the nucleon-nucleon interaction. This new amplitude provides a superior description of $pp\gamma$ data. The predictions of this new amplitude are in close agreement with potential-model calculations, which implies that, contrary to conclusions drawn by others, off-shell effects are essentially insignificant below pion-production threshold.

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Bremsstrahlung processes have been used as a tool to investigate electromagnetic properties of resonances, details of reaction mechanisms, and off-shell properties of scattering amplitudes. The most succesful example in the first case is the determination of the magnetic moments of the Δ^{++} (Δ^0) from $\pi^+ p\gamma$ ($\pi^- p\gamma$) data in the energy region of the $\Delta(1232)$ resonance [1]. In the case of reaction mechanisms, a well-known example is the extraction of nuclear time delays from the $p^{12}C\gamma$ data near the 1.7-MeV resonance [2]. The time delay distinguishes between direct and compound nuclear reactions. The initial goal of nucleon-nucleon bremsstrahlung investigations was to distinguish among various phenomenological potential models of the fundamental two-nucleon interaction. Most measured $pp\gamma$ cross sections could, in fact, be reasonably described by potential-model calculations, but the difference between predictions from any two realistic potentials appears to be too small to be distinguished by the data.

For more than 30 years, the conventional Low soft-photon amplitude [3] has been widely used for studying nuclear and particle bremsstrahlung processes. It seemingly provides a good description of the data for some processes. For instance, Nyman [4] and Fearing [5] used this amplitude to calculate $pp\gamma$ cross sections which were in reasonable agreement with several measurements and potential-model calculations. However, it was recently pointed out by Workman and Fearing [6] that the results from this conventional Low amplitude differ significantly from the potential-model calculations for the TRIUMF data at 280 MeV [7]. This difference was interpreted as evidence for "off-shell effects" in the $pp\gamma$ process.

The main purpose of this Letter is to propose a novel soft-photon amplitude to replace the conventional Low prescription. This new amplitude, the derivation of which is guided by the structure of the standard meson-exchange model of the two-nucleon interaction, is relativistic, manifestly gauge invariant, and consistent with the soft-photon theorem. It belongs to one of the two general classes of recently derived soft-photon amplitudes [8]. We demonstrate that the $pp\gamma$ data from low energies to energies near the pion-production threshold can be consistently described by the new amplitude. Most importantly, we point out here that our amplitude essentially eliminates the discrepancy between the soft-photon

approximation and the potential-model calculations. That is, we demonstrate that “off-shell effects” are essentially negligible. Finally, we explore why the conventional Low amplitude works for some cases but fails for others.

In order to elucidate these points, let us consider photon emission accompanying the scattering of two spin-1/2 particles A and B,

$$A(q_i^\mu) + B(p_i^\mu) \rightarrow A(q_f^\mu) + B(p_f^\mu) + \gamma(K^\mu) \ . \tag{1}$$

Here, q_i^μ (q_f^μ) and p_i^μ (p_f^μ) are the initial (final) four-momenta for particles A and B, respectively, and K^μ is the four-momentum for the emitted photon with polarization ε^μ. Particle A (B) is assumed to have mass m_A (m_B), charge Q_A (Q_B), and anomalous magnetic moment κ_A (κ_B). For process (1), we can define the following Mandelstam variables: $s_i = (q_i + p_i)^2$, $s_f = (q_f + p_f)^2$, $t_q = (q_f - q_i)^2$, $t_p = (p_f - p_i)^2$, $u_1 = (p_f - q_i)^2$, and $u_2 = (q_f - p_i)^2$. Since a soft-photon amplitude depends only on either (s,t) or (u,t), chosen from the above set, we can derive two distinct classes of soft-photon amplitudes: $M_\mu^{(1)}(s,t)$ and $M_\mu^{(2)}(u,t)$ [8]. The general amplitude from the first class is the two-s–two-t special amplitude $M_\mu^{TsTts}(s_i, s_f; t_q, t_p)$; that from the second class is the two-u–two-t special amplitude $M_\mu^{TuTts}(u_1, u_2; t_q, t_p)$. The distinguishing characteristics of these amplitudes come from the fact that they are evaluated at different elastic-scattering or on-shell points (energy and angle). The soft-photon theorem does not specify how these on-shell points are to be selected.

The modified procedure for deriving these soft-photon amplitudes is described in detail in Ref. [8]. In this procedure, the fundamental tree diagrams of the underlying elastic scattering process play an important role in deriving the two general amplitudes. Thus, we argue that M_μ^{TsTts} should be used to describe those processes which are resonance dominated [such as $p^{12}C\gamma$ near 1.7 MeV and $\pi^\pm p\gamma$ in the $\Delta(1232)$ region], whereas M_μ^{TuTts} should be used to describe those processes which are exchange-current dominated (such as the $np\gamma$ process). For the $pp\gamma$ process, which exhibits neither strong resonance effects nor significant u-channel exchange-current effects, both amplitudes can be used in theory, although this has never been tested in conjunction with experimental data. We provide here the results

of such an analysis. We emphasize that the general amplitude M_μ^{TuTts} (not M_μ^{TsTts}) arises naturally for nucleon-nucleon bremsstrahlung *if* the derivation is guided by the standard meson-exchange model of the two-nucleon interaction.

The amplitude M_μ^{TuTts} for the $pp\gamma$ process can be written in terms of five invariant amplitudes F_α^e $(\alpha = 1, \ldots, 5)$ as

$$M_\mu^{TuTts} = \sum_{\alpha=1}^{5} \left[Q_A \overline{u}(q_f) X_{\alpha\mu} u(q_i) \overline{u}(p_f) g^\alpha u(p_i) + Q_B \overline{u}(q_f) g_\alpha u(q_i) \overline{u}(p_f) Y_\mu^\alpha u(p_i) \right] \ , \tag{2}$$

where

$$\begin{aligned} X_{\alpha\mu} &= F_\alpha^e(u_1, t_p) \left[\frac{q_{f\mu} + R_\mu^{q_f}}{q_f \cdot K} - \frac{(p_i - q_f)_\mu}{(p_i - q_f) \cdot K} \right] g_\alpha \\ &\quad - F_\alpha^e(u_2, t_p) g_\alpha \left[\frac{q_{i\mu} + R_\mu^{q_i}}{q_i \cdot K} - \frac{(q_i - p_f)_\mu}{(q_i - p_f) \cdot K} \right] \ , \end{aligned} \tag{3}$$

$$\begin{aligned} Y_\mu^\alpha &= F_\alpha^e(u_2, t_q) \left[\frac{p_{f\mu} + R_\mu^{p_f}}{p_f \cdot K} - \frac{(q_i - p_f)_\mu}{(q_i - p_f) \cdot K} \right] g^\alpha \\ &\quad - F_\alpha^e(u_1, t_q) g^\alpha \left[\frac{p_{i\mu} + R_\mu^{p_i}}{p_i \cdot K} - \frac{(p_i - q_f)_\mu}{(p_i - q_f) \cdot K} \right] \ . \end{aligned} \tag{4}$$

In Eqs. (2-4), we have defined

$$(g_1, g_2, g_3, g_4, g_5) \equiv (1, \sigma_{\mu\nu}/\sqrt{2}, i\gamma_5\gamma_\mu, \gamma_\mu, \gamma_5) \ ,$$

$$(g^1, g^2, g^3, g^4, g^5) \equiv (1, \sigma^{\mu\nu}/\sqrt{2}, i\gamma_5\gamma^\mu, \gamma^\mu, \gamma_5) \ ,$$

and the factors R_μ^Q $(Q = q_f, q_i, p_f, p_i)$ can be expressed as

$$R_\mu^Q = \frac{1}{4} [\gamma_\mu, \not{K}] + \frac{\kappa}{8m} \{ [\gamma_\mu, \not{K}], \not{Q} \} \ . \tag{5}$$

In Eq. (5), m $(= m_A = m_B)$ and κ $(= \kappa_A = \kappa_B)$ are the mass and the anomalous magnetic moment of the proton, $\not{Q} = Q^\mu \gamma_\mu$, and we have used $[F, G] \equiv FG - GF$ and $\{F, G\} \equiv FG + GF$. As one can see from Eqs. (3) and (4), the invariant amplitudes F_α^e depend on u and t. The same amplitudes but as functions of s and t can be obtained if we use the condition $s + t + u = 4m^2$. For example, $F_\alpha^e(u_1, t_p) = F_\alpha^e(s_{1p}, t_p)$ where $s_{1p} + t_p + u_1 = 4m^2$. Since $F_\alpha^e(s_{1p}, t_p)$ $(\alpha = 1, \ldots, 5)$ are invariant amplitudes for the pp elastic process, the

Feynman amplitude $F(s_{1p}, t_p)$ defined by Goldberger *et al.* [9] can be written in terms of the five Fermi covariants (S, T, A, V, P) as

$$F(s_{1p}, t_p) = F_1^e(s_{1p}, t_p)S + F_2^e(s_{1p}, t_p)T + F_3^e(s_{1p}, t_p)A + F_4^e(s_{1p}, t_p)V + F_5^e(s_{1p}, t_p)P \ . \quad (6)$$

The amplitude $M_\mu^{TsTts}(s_i, s_f; t_q, t_p)$ can be formally obtained from the amplitude $M_\mu^{TuTts}(u_1, u_2; t_q, t_p)$ given by Eqs. (2), (3), and (4) by making the following substitutions: (i) $Q_B \to -Q_B$ and (ii) $p_i^\mu \leftrightarrow -p_f^\mu$ and $g^\alpha R_\mu^{p_i} \leftrightarrow -R_\mu^{p_f} g^\alpha$, keeping $R_\mu^{q_i}$, $R_\mu^{q_f}$, and the spinors $\overline{u}$ and u unchanged. However, we emphasize that the two are not the same numerically.

If all $F_\alpha^e(s_x, t_y)$ $(\alpha = 1, \ldots, 5,\ x = i, f,$ and $y = q, p)$ in M_μ^{TsTts} are expanded about average s, $\overline{s}$, and average t, $\overline{t}$, then the first two terms of the expansion give the conventional Low amplitude $M_\mu^{\mathrm{Low}(s,t)}(\overline{s}, \overline{t})$. This particular choice $(\overline{s}, \overline{t})$ for the on-shell point at which the Low amplitude is evaluated is just an *ad hoc* prescription, although it provided a reasonable description of $pp\gamma$ data until the TRIUMF measurements at 280 MeV.

We have studied the amplitudes M_μ^{TuTts}, M_μ^{TsTts}, and $M_\mu^{\mathrm{Low}(s,t)}$ and have applied them to calculate $pp\gamma$ cross sections at various energies, using state-of-the-art phase shifts from the latest Nijmegen pp partial-wave analysis [10]. Anecdotal results are shown in Figs. 1, 2, and 3. At 42 MeV for $\theta_q = \theta_p = 26°$ (see Fig. 1) the coplanar cross sections calculated from M_μ^{TsTts} are much larger than the Manitoba data [11]. The amplitudes M_μ^{TuTts} and $M_\mu^{\mathrm{Low}(s,t)}$, on the other hand, give similar results which agree well with both the data (within the experimental error) and the representative Hamada-Johnston-potential calculation [12]. The results calculated using $M_\mu^{\mathrm{Low}(s,t)}$ are close to those obtained by Nyman and Fearing. In Fig. 2 our coplanar cross sections calculated from M_μ^{TuTts} and $M_\mu^{\mathrm{Low}(s,t)}$ at 157 MeV for $\theta_q = \theta_p = 35°$ are compared with the Harvard data [13] and a Paris-potential calculation [14]. (Other potential-model calculations [6,15–17] which include relativistic spin-corrections etc. are similar.) Cross sections calculated using the amplitude M_μ^{TsTts} are missing from Figs. 2 and 3, because they are factors larger than those plotted. Again the amplitudes M_μ^{TuTts} and $M_\mu^{\mathrm{Low}(s,t)}$ give very similar results at this energy and agree reasonably with both the potential-model curve and the Harvard data.

However, at an energy near the pion-production threshold and far from the on-shell point, the two amplitudes M_μ^{TuTts} and $M_\mu^{\mathrm{Low}(s,t)}$ predict quite different results. This is demonstrated in Fig. 3. At 280 MeV for $\theta_q = 12.4^\circ$ and $\theta_p = 12^\circ$, the curve calculated from M_μ^{TuTts} agrees well with the published TRIUMF data [7] and with the curves calculated using the Paris potential and the Bonn potential [7]. The amplitude $M_\mu^{\mathrm{Low}(s,t)}$, on the other hand, predicts cross sections which are too small for forward ($\theta_\gamma \leq 30^\circ$) and backward ($\theta_\gamma \geq 150^\circ$) photon angles. That $M_\mu^{\mathrm{Low}(s,t)}$ can describe most of the older $pp\gamma$ data but fails to fit the new TRIUMF data has already been pointed out by Fearing. What is emphasized here is that the new amplitude M_μ^{TuTts} describes data where the conventional Low amplitude $M_\mu^{\mathrm{Low}(s,t)}$ fails. In other words, the correct soft-photon amplitude which describes the $pp\gamma$ data consistently is M_μ^{TuTts}.

How can we understand the failure of the conventional Low amplitude $M_\mu^{\mathrm{Low}(s,t)}$? Consider the expressions given in Eqs. (2-4). If we impose the on-shell condition, $s+t+u = 4m^2$, we can write $F_\alpha^e(u_1, t_p) = F_\alpha^e(s_{1p}, t_p)$, $F_\alpha^e(u_2, t_p) = F_\alpha^e(s_{2p}, t_p)$, $F_\alpha^e(u_1, t_q) = F_\alpha^e(s_{1q}, t_q)$, and $F_\alpha^e(u_2, t_q) = F_\alpha^e(s_{2q}, t_q)$, where $s_{1p} = s_i - 2q_f \cdot K$, $s_{2p} = s_i - 2p_i \cdot K$, $s_{2q} = s_i - 2p_f \cdot K$, and $s_{1q} = s_i - 2q_i \cdot K$. This shows that F_α^e will be evaluated at four different energies and four different angles in constructing M_μ^{TuTts}. (Potential-model calculations also use four-energy-four-angle amplitudes.) In contrast, M_μ^{TsTts} is evaluated at two energies and four angles, while $M_\mu^{\mathrm{Low}(s,t)}$ is evaluated at just one energy and one angle. To be specific, at 100 MeV for $\theta_q = \theta_p = \theta_\gamma = 30^\circ$, we have $s_{1p} = 3.648$ GeV^2, $s_{2q} = 3.640$ GeV^2, $s_{2p} = 3.632$ GeV^2, and $s_{1q} = 3.655$ GeV^2, whereas $s_i = 3.709$ GeV^2 and $s_f = 3.578$ GeV^2, and finally $\overline{s} = 3.644$ GeV^2. These quantities are the dominant factors determining the calculated cross sections. Since $s_{1p} \simeq s_{2p} \simeq s_{2q} \simeq s_{1q} \simeq \overline{s}$ (the differences in c.m. energy between s_{1p}, s_{2p}, s_{2q}, and s_{1q} on one hand, and $\overline{s}$ on the other hand, are less than about 3 MeV), $M_\mu^{\mathrm{Low}(s,t)}$ and M_μ^{TuTts} predict similar results at energies lower than 100 MeV and for large proton angles. However, the value of s_i is much larger than the value of s_f. This is equivalent to a c.m. energy difference of some 34 MeV. This large difference between s_i and s_f is the primary reason

for the huge cross sections predicted by M_μ^{TsTts}. As the incident energy increases (or the proton angles decrease), the values of the four energies, s_{1p}, s_{2p}, s_{1q}, and s_{2q}, will no longer be close to one another, and they differ significantly from $\overline{s}$, as well as s_i and s_f. Thus, the cross sections calculated using the amplitude M_μ^{TuTts} will differ from those calculated using either $M_\mu^{\mathrm{Low}(s,t)}$ or M_μ^{TsTts}. A more systematic analysis, including other relevant factors, will be given elsewhere.

In conclusion, we have demonstrated that the amplitude M_μ^{TuTts}, not the conventional Low amplitude $M_\mu^{\mathrm{Low}(s,t)}$ nor the amplitude M_μ^{TsTts}, is the correct soft-photon amplitude to be used in describing the nucleon-nucleon bremsstrahlung processes. Furthermore, below the pion-production threshold this novel amplitude M_μ^{TuTts} provides a description of the $pp\gamma$ data that is the equal of contemporary potential-model calculations, implying off-shell effects are insignificant in the kinematic range measured to date.

ACKNOWLEDGMENTS

We would like to thank M. Rentmeester for providing the phase shifts from the latest Nijmegen proton-proton partial-wave analysis. The work of M.K.L. was supported in part by the City University of New York Professional Staff Congress-Board of Higher Education Faculty Research Award Program, while the work of R.T. and B.F.G. was performed under the auspices of the Department of Energy.

REFERENCES

[1] Dahang Lin, M.K. Liou, and Z.M. Ding, Phys. Rev. C **44**, 1819 (1991), and references therein; M.K. Liou and Dahang Lin (unpublished).

[2] C. Maroni, I. Massa, and G. Vannini, Nucl. Phys. **A273**, 429 (1976); C.K. Liu, M.K. Liou, C.C. Trail, and P.M.S. Lesser, Phys. Rev. C **26**, 723 (1982); H. Taketani, M. Adachi, N. Endo, and T. Suzuki, Phys. Lett. **113B**, 11 (1982).

[3] F.E. Low, Phys. Rev. **110**, 974 (1958).

[4] E.M. Nyman, Phys. Rev. **170**, 1628 (1968).

[5] H.W. Fearing, Phys. Rev. C **22**, 1388 (1980).

[6] R.L. Workman and H.W. Fearing, Phys. Rev. C **34**, 780 (1986).

[7] K. Michaelian *et al.*, Phys. Rev. C **41**, 2689 (1990).

[8] M.K. Liou, D. Lin, and B.F. Gibson, Phys. Rev. C **47**, 973 (1993).

[9] M.L. Goldberger, M.T. Grisaru, S.W. MacDowell, and D.Y. Wong, Phys. Rev. **120**, 2250 (1960).

[10] J.R. Bergervoet, P.C. van Campen, R.A.M. Klomp, J.-L. de Kok, T.A. Rijken, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **41**, 1435 (1990); V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C **48**, 792 (1993).

[11] J.V. Jovanovich *et al.*, Phys. Rev. Lett. **26**, 277 (1971).

[12] M.K. Liou and M.I. Sobel, Ann. Phys. (NY) **72**, 323 (1972).

[13] B. Gottschalk, W.J. Shlaer, and K.H. Wang, Nucl. Phys. **A94**, 491 (1967).

[14] M. Jetter, H. Freitag, and H.V. von Geramb, Phys. Scr. **48**, 229 (1993).

[15] V. Herrmann and K. Nakayama, Phys. Rev. C **45**, 1450 (1992).

[16] V.R. Brown, P.L. Anthony, and J. Franklin, Phys. Rev. C **44**, 1296 (1991).

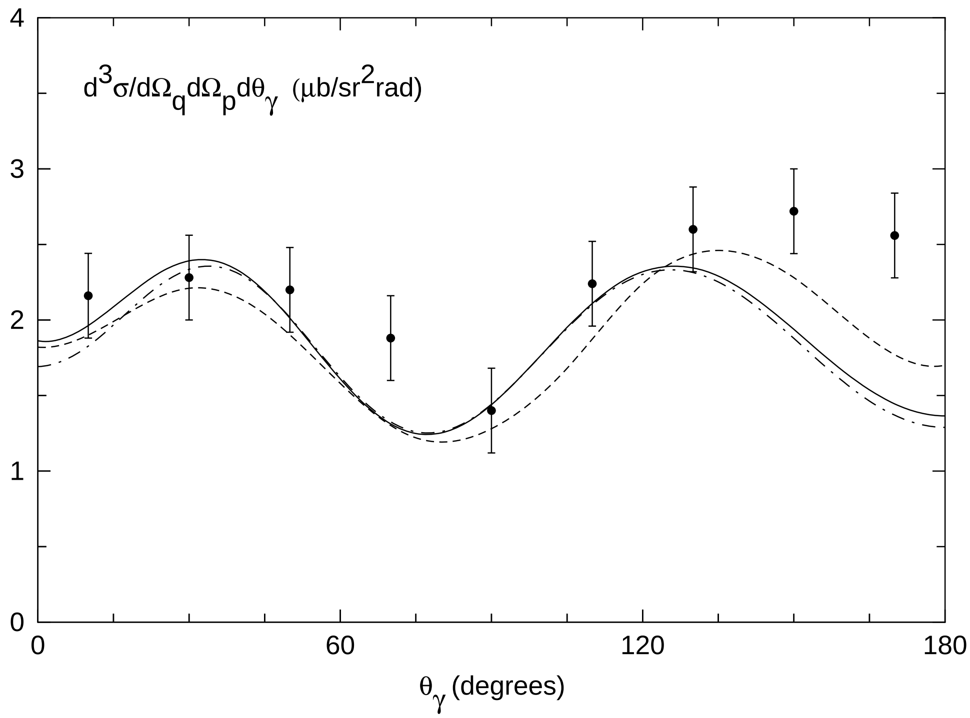
[17] A. Katsogiannis and K. Amos, Phys. Rev. C **47**, 1376 (1993).

FIGURES

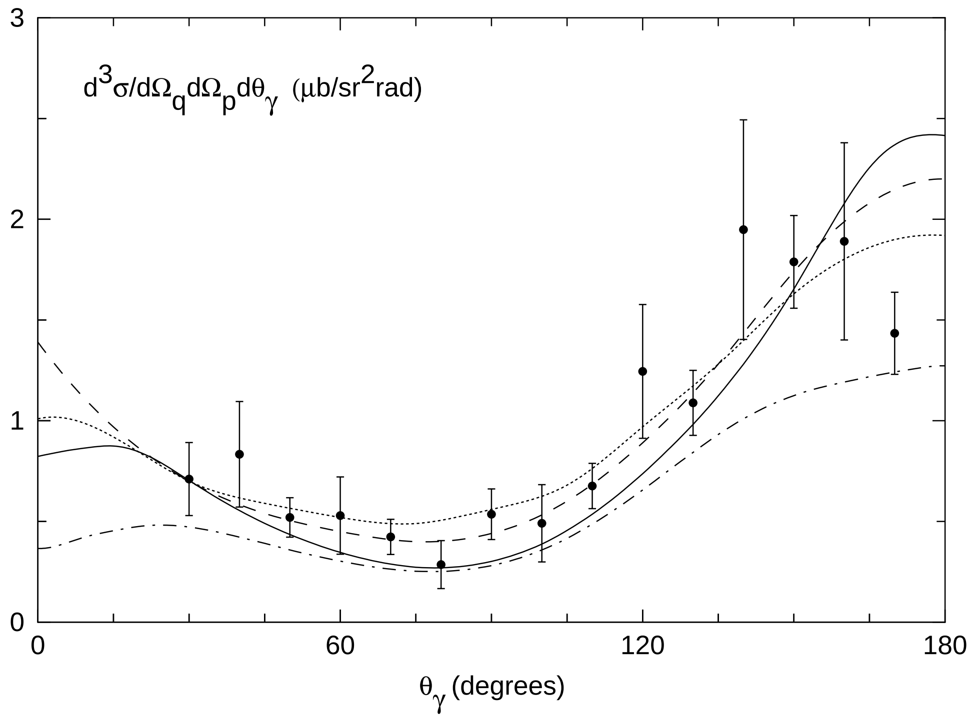
FIG. 1. Coplanar $pp\gamma$ cross section at 42 MeV for $\theta_q = \theta_p = 26°$; ——: result using M_μ^{TuTts}; $- \cdot - \cdot$: result using $M_\mu^{\mathrm{Low}(s,t)}$; $- - -$: result using M_μ^{TsTts}; $\cdots\cdots$: result for Hamada-Johnston potential [12]. The data are from Ref. [11].

FIG. 2. Coplanar $pp\gamma$ cross section at 157 MeV for $\theta_q = \theta_p = 35°$; ——: result using M_μ^{TuTts}; $- \cdot - \cdot$: result using $M_\mu^{\mathrm{Low}(s,t)}$; $- - -$: result for Paris potential [14]. The data are from Ref. [13].

FIG. 3. Coplanar $pp\gamma$ cross section at 280 MeV for $\theta_q = 12.4°, \theta_p = 12°$; ——: result using M_μ^{TuTts}; $- \cdot - \cdot$: result using $M_\mu^{\mathrm{Low}(s,t)}$; $- - -$: result for Paris potential [7]; $\cdots\cdots$: result for Bonn potential [7]. The data are from Ref. [7].



$d^3\sigma/d\Omega_q d\Omega_p d\theta_\gamma$ (μb/sr^{2}rad)
4
3
2
1
0
0
60
120
180
θ_γ (degrees)



3
2
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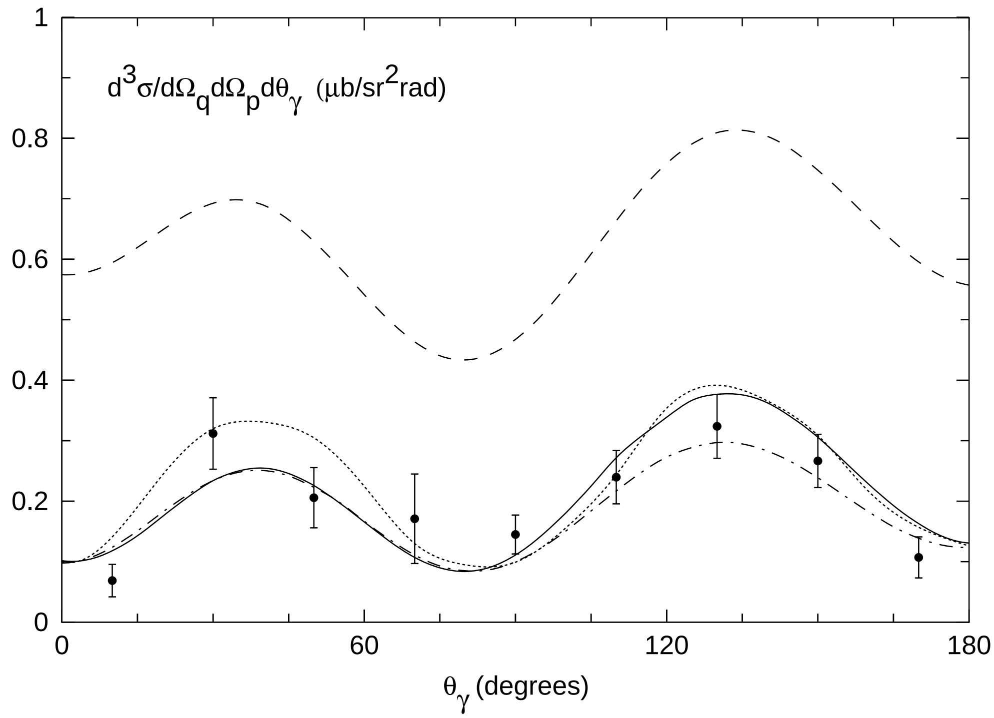
$d^3\sigma/d\Omega_q d\Omega_p d\theta_\gamma$ (μb/sr^{2}rad)
0
60
120
180
θ_γ (degrees)



1
0.8
0.6
0.4
0.2
0
0
60
120
180
$d^3\sigma/d\Omega_q d\Omega_p d\theta_\gamma$ (μb/sr^{2}rad)
θ_γ (degrees)